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Reversing a Truck-Full Trailer Considering the Feedback Delay

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In this paper, controlling the reverse motion along a straight-line in case of a truck-full trailer is investigated. The single track kinematic model is applied to analyze the vehicle system taking into account the feedback delay and the dynamics of the steering mechanism. Linear stability charts are calculated to determine the appropriate control gains of the applied linear state feedback controller. Furthermore, vibration frequencies related to the stability boundaries are also presented. The obtained results are verified and validated via simulations and experiments using a small-scale experimental test rig.

Topics / Autonomous Driving Systems, Advanced Driver Assistance Systems

1. INTRODUCTION

Reversing a car-trailer combination is a complicated task for human drivers. Hence, Driver Assistance Systems (DAS) are designed and commercialized to facilitate reversing maneuvers for inexperienced drivers [1, 2], however, these systems only provide support in reversing maneuvers, but the main controller is still the driver [3].

Although in the field of autonomous road vehicles most companies deal with self-driving features of passenger cars, the case of trucks is at least as important. Let us mention the importance of reducing fuel consumption and delivery time in freight transport. These tasks could be accomplished by forming a chain of trucks in which only the head vehicle has a human driver. Applying driverless control in the follower vehicles shortens the headway, thereby reducing air resistance. Another important application is realizing complicated maneuvers with long vehicles, such as reversing into the loading dock.

The development of self-driving trucks requires motion control solutions similar to cars, but they have to be able to handle more difficult scenarios as well. For example, in case of a truck-full trailer combination, two articulated joints are included in the vehicle system, and the reversing motion is even more complicated. Reaching exponential stability of the forward motion is known even in case of a chain of trailers attached to a car [4], but reversing this assembly is a different task. The reverse motion itself is unstable without active control in any condition. However, the practical realization of reversing a trailer using DAS is available, see [5]. Although the reversing motion of trailers has been investigated for several decades, see, for example [6, 7, 8], the topic is still actively researched nowadays, see [9].

The main purpose of this paper is to create a simple control algorithm which can ensure the reverse motion of the truck-full trailer combination along a straight path even in the presence of non-negligible feedback delay. The applicability of this control method is supported by simulations, furthermore, the results are validated via tests using a small-scale experimental rig.

2. MECHANICAL MODEL

The vehicle system is analyzed using the single track kinematic model presented in Fig. 1. The model consists of three parts (modeled with three rods), denoted by the encircled numbers in the figure: the towing vehicle is marked with 1; 2 denotes the drawbar creating the coupling between the truck and the trailer (so-called dolly); and 3 marks the trailer. During the calculations, the longitudinal speed of the rear axle of the towing truck is assumed to be a constant value V.

The equations of motion can be expressed by five generalized coordinates: X_R, Y_R gives the position of the rear axle center point R of the truck; ψ_1 is the yaw angle of the truck; φ_2 and φ_3 denote the relative yaw angles of the dolly and the trailer respectively. The steering angle of the front wheel of the truck is marked with δ. The

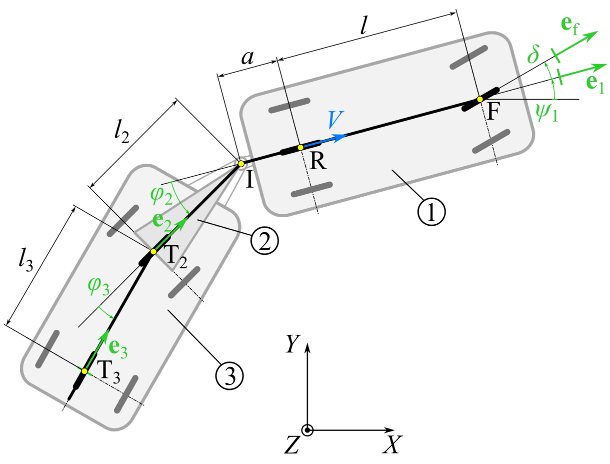

Fig. 1 Mechanical model

actuation of the vehicle system is realized via the steering of the front wheel of the truck. The dynamics of the steering mechanism is also considered in our study, and we also put emphasis on taking into account the time delay of the controller. Finally, the equations of motion can be obtained as

$$\begin{gathered}\dot{X} = V\cos\psi_1, \quad \dot{Y} = V\sin\psi_1, \quad \dot{\psi}_1 = \tfrac{V}{l}\tan\delta, \\ \dot{\varphi}_2 = -\tfrac{V}{l l_2}(l\sin\varphi_2 + (l_2 + a\cos\varphi_2)\tan\delta), \\ \dot{\varphi}_3 = -\tfrac{1}{l_3}\big(V\sin(\varphi_2+\varphi_3) + a\dot{\psi}_1\cos(\varphi_2+\varphi_3) \\ + l_2(\dot{\psi}_1+\dot{\varphi}_2)\cos\varphi_2\big) - (\dot{\psi}_1+\dot{\varphi}_2),\end{gathered} \tag{1}$$

where the undetailed parameters (a, l, l_2, l_3) are geometric parameters defined in Fig. 1.

3. CONTROLLER

In this study, the stable reverse motion is achieved by a linear feedback controller with three proportional terms. Accordingly, the control law is as follows:

$$\begin{aligned}\delta_{\mathrm{des}}(t) = -P_Y Y(t-\tau) - P_{\psi_1}\psi_1(t-\tau) \\ - P_{\varphi_3}\varphi_3(t-\tau),\end{aligned} \tag{2}$$

where τ is the feedback delay, P_Y, $P_{\psi1}$ and $P_{\varphi3}$ are the control gains. It is important to emphasize that the relative yaw angle φ_2 of the drawbar is not required to stabilize the straight-line reverse motion. Note, measuring of this angle could be a difficult task in practice, especially in cases when trailers are changed.

In addition to the feedback control, there is a lower-level controller to operate the steering mechanism, which can be written as

$$\dot{\delta} = \omega, \qquad \dot{\omega} = -\frac{k_{\mathrm{p}}}{J}(\delta - \delta_{\mathrm{des}}) - \frac{k_{\mathrm{d}}}{J}\omega, \tag{3}$$

where J is the mass moment of inertia of the steering system, k_{p} and k_{d} are the gains of the lower-lever controller. Attaching Eq. (3) to Eq. (1) gives the equations of motion to the overall system. The controllability and the observability of the system was also proved with the three terms, i.e. without the feedback of the relative yaw angle φ_2 of the drawbar (dolly).

4. STABILITY

The linear stability of the reverse motion along a straight path is analyzed using both the D-subdivision and the semi-discretization methods. Thus, stability charts are constructed to locate the domain of control gains ensuring stable reversing motion.

The effect of time delay on stability is shown in Fig. 2, where stability boundaries (black curves determined by the D-subdivision method) are plotted in the plane of the higher level control gains $P_{\psi1}$ and $P_{\varphi3}$ for the parameters of a small-scale model. The control gain related to the lateral position is fixed at P_Y = 5 rad/m, while the longitudinal speed is V = –0.1 m/s (the negative sign corresponds to the reverse direction). Linearly stable domains are shaded based on the semi-discretization. Darker domains refer to larger delays. As shown in the

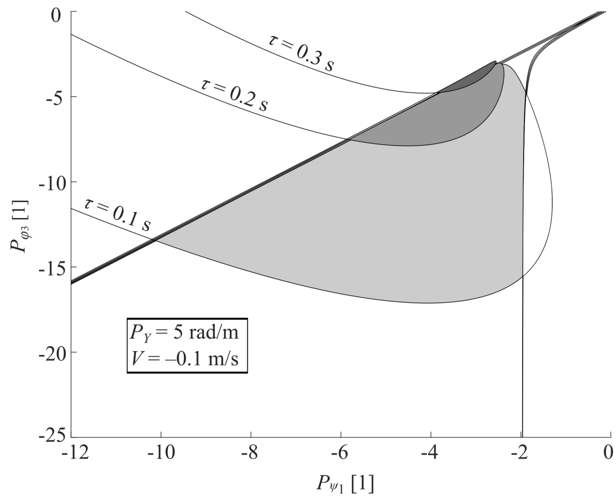

Fig. 2 Effect of time delay on stability. Different shades of gray relate to different value of time delay. Parameter values: a = 0.05 m, l = 0.24 m, l_2 = 0.09 m, l_3 = 0.255 m, $J = 10^{-6}$ kgm^2, $k_{\mathrm{p}} = 3\cdot10^{-4}$ Nm, $k_{\mathrm{d}} = 1.7321\cdot10^{-5}$ kgm^2/s

figure, time delay has a significant impact on stability. The larger the delay is, the smaller the stable domain is.

In more detail, stability boundaries determined by the two different methods are compared in Fig. 3. Black lines refer to the D-curves which are determined analytically and they are possible stability boundaries, and the red dashed curve is the result obtained by the semi-discretization method, which appoints the real stable domain. The vibration frequencies related to the curves are determined by the analytic analysis of the D-curves. This gives the numerical values (related to the angular frequency ω) shown at specific points of the stability boundary. Crossing the border at a particular point leads to loss of stability with oscillations emerging at the current frequency. The point marked by HH notation is a possible double-Hopf bifurcation point. Crossing the boundary at this point causes a multi-component frequency oscillation, accordingly, this point has two different frequency values.

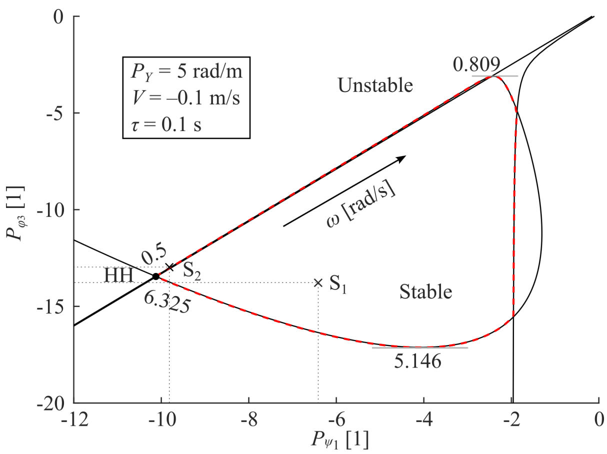

Fig. 3 Stability chart in the plane of control gains $P_{\psi1}$ and $P_{\varphi3}$ using both the D-subdivision (black continuous line) and the semi-discretization method (red dashed line). Numbers on graph refer to frequency analysis

Note that the model parameters and configuration used here refer to the proper experimental rig on which tests are performed and will be introduced later in this paper.

5. SIMULATIONS AND EXPERIMENTS

In order to make sure that the calculations are right and to visualize the motion, simulations are carried out. Finally, experimental tests validate the theoretical results. The geometric parameters applied to the simulations are the same as those used in the stability calculations.

5.1 Simulations

Simulations are shown in Fig. 4 and Fig. 5, where time series appear related to the lateral position Y, the yaw angle ψ_1 of the truck, the relative yaw angle φ_3 of the trailer and the steering angle δ, respectively. These simulations are started with an initial condition $Y(t=0) = 0.05$ m, which corresponds to a lane changing maneuver. More precisely, this initial condition is an error for the controller to be reduced, so the stability of the motion depends on the magnitude of that initial value. It is worth noting that if the perturbation is too large (in our case: >0.1 m), the motion cannot be stabilized.

In Fig. 4, simulation at the most stable gain-configuration (where the rightmost eigenvalue has the smallest real part) is shown. Accordingly, the controller ensures stability of the rectilinear reverse motion, but significant oscillations of the steering input can be observed. The simulation in Fig. 5 corresponds to a configuration picked from the vicinity of the HH point. Two frequencies can be discovered as it is expected from the analytical results. The frequency values identified in the time series agree with the results of the frequency analysis (see Fig. 3), i.e., $\omega_1 = 0.5$ rad/s ≈ 0.08 Hz, and $\omega_2 = 6.325$ rad/s ≈ 1 Hz. These vibrations can be recognized in the time series of the yaw angle ψ_1 (Fig. 5), which means approximately 2.5 periods in case of ω_1 and 30 periods in case of ω_2. The related gain-configurations are marked with black crosses labeled with S_1 (most stable case) and S_2 (double-Hopf bifurcation case) in the stability chart in Fig. 3.

5.2. Experiments

In our study, the theoretical results are also validated via experimental tests. For this, we use the small-scale experimental test rig shown in Fig. 6. The lateral position and yaw angle of the towing vehicle on the treadmill is measured via a linear encoder and magnetic rotary sensors. The relative angle between the trailer and the dolly is also measured by a magnetic rotary sensor. The control algorithm is implemented into an NI cRio system, which, in possession of the sensors data, can calculate the desired steering angle and control the servo mechanism in real time.

The stability chart for the small-scale test rig scenario (i.e. all parameters are the same as for the simulations), as a major result of our study, is shown in Fig. 7. The theoretical stable domain is depicted with gray area. As can be seen, the straight-line reverse motion can be stabilized for this low speed reversing even in case of relatively large feedback delay.

Measurements were carried out by setting different control gain setups and the stability properties were

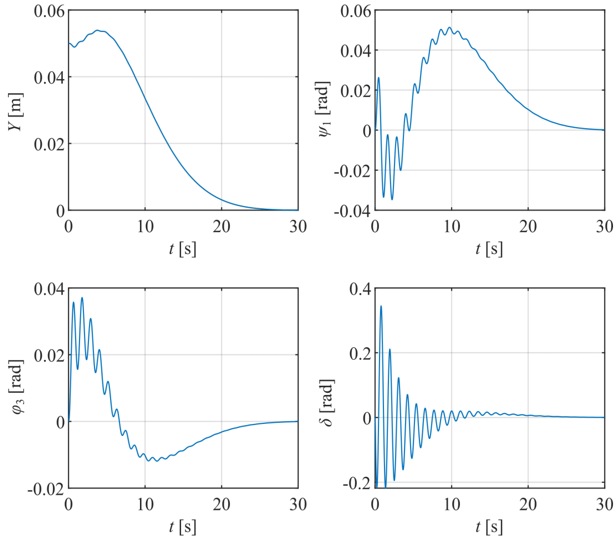

Fig. 4 Simulation results for the most stable gain-configuration (point S_1). Control gains: $P_Y = 5$ rad/m, $P_{\psi 1} = -6.421$, $P_{\varphi 3} = -13.82$. Initial condition: $Y(t=0) = 0.05$ m

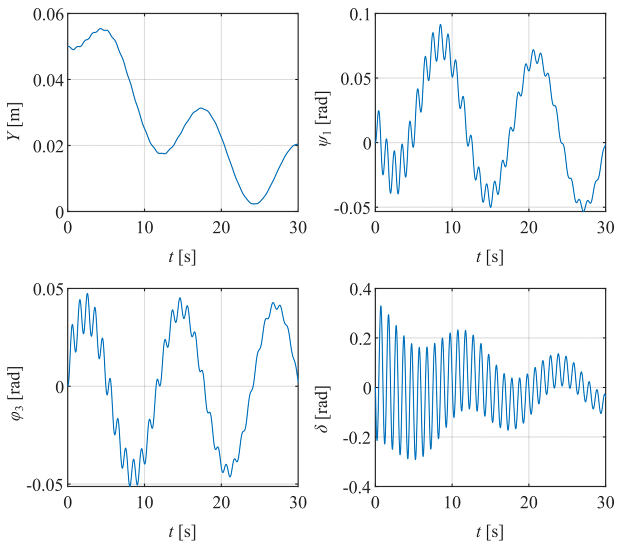

Fig. 5 Simulation results for a gain-configuration (point S_2) picked from near the double-Hopf bifurcation point. Control gains: $P_Y = 5$ rad/m, $P_{\psi 1} = -9.8$, $P_{\varphi 3} = -13$. Initial condition: $Y(t=0) = 0.05$ m

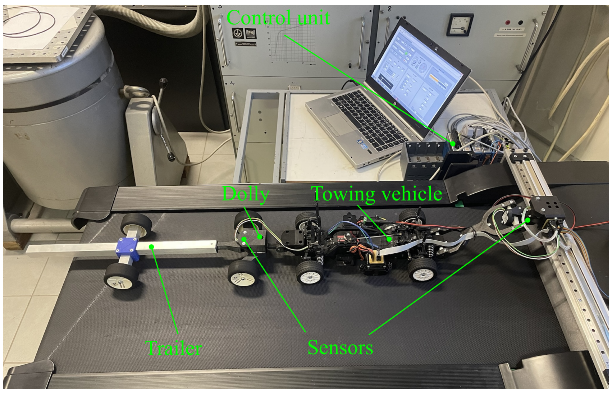

Fig. 6 Photo of the small-scale experimental test rig

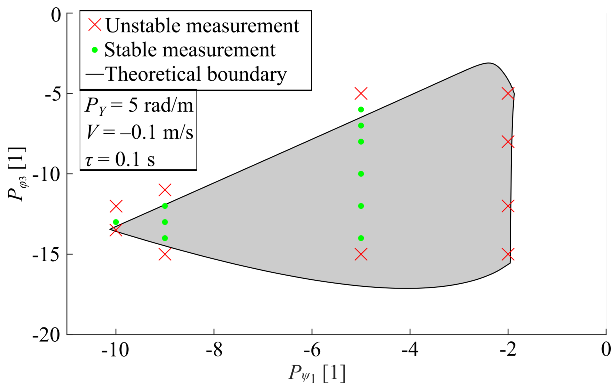

Fig. 7 Comparison of theoretical and experimental results: linear stability boundary (black curve) in the plane of control gains $P_{\psi 1}$ and $P_{\varphi 3}$ for the parameters of the small-scale experimental rig (grey domain), and the measurement results (green dots and red crosses)

identified. It was achieved by changing the reference line (i.e. causing a lane change maneuver) as perturbation and deciding whether the motion is stable or unstable. The results are depicted in Fig. 7 by green dots and red crosses. Good agreement can be established between the theoretical and measured stability limits.

6. CONCLUSION

A linear feedback controller with three proportional gains is designed to maintain the reverse motion of a truck-full trailer along a straight-line. This paper shows that such a vehicle system can be stabilized by feedback of the lateral position and the yaw angle of the towing vehicle, and the yaw angle of the trailer relative to the dolly. The kinematic model is supplemented with the dynamics of the steering mechanism. Feedback delay is also considered, which has a significant effect on stability. Non-linear simulations were also carried out, thus, the stability charts and the frequency analysis based on the D-subdivision method were verified. The results are validated via experimental tests using a small-scale experimental test rig.

Realizing the path-following feature in reverse motion is intended in our further studies. As it turned out from both the simulations and the experimental tests, only a small perturbation can be allowed to maintain stability, hence, general path-following is essential in order to perform complicated maneuvers with the truck-full trailer. The case of reversing along a straight line presented in this paper provides a basis for developing path-following control. Later, this feature could be implemented in the industry, supporting truck drivers in maneuvering in the loading bay, thus reducing the loading time.

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